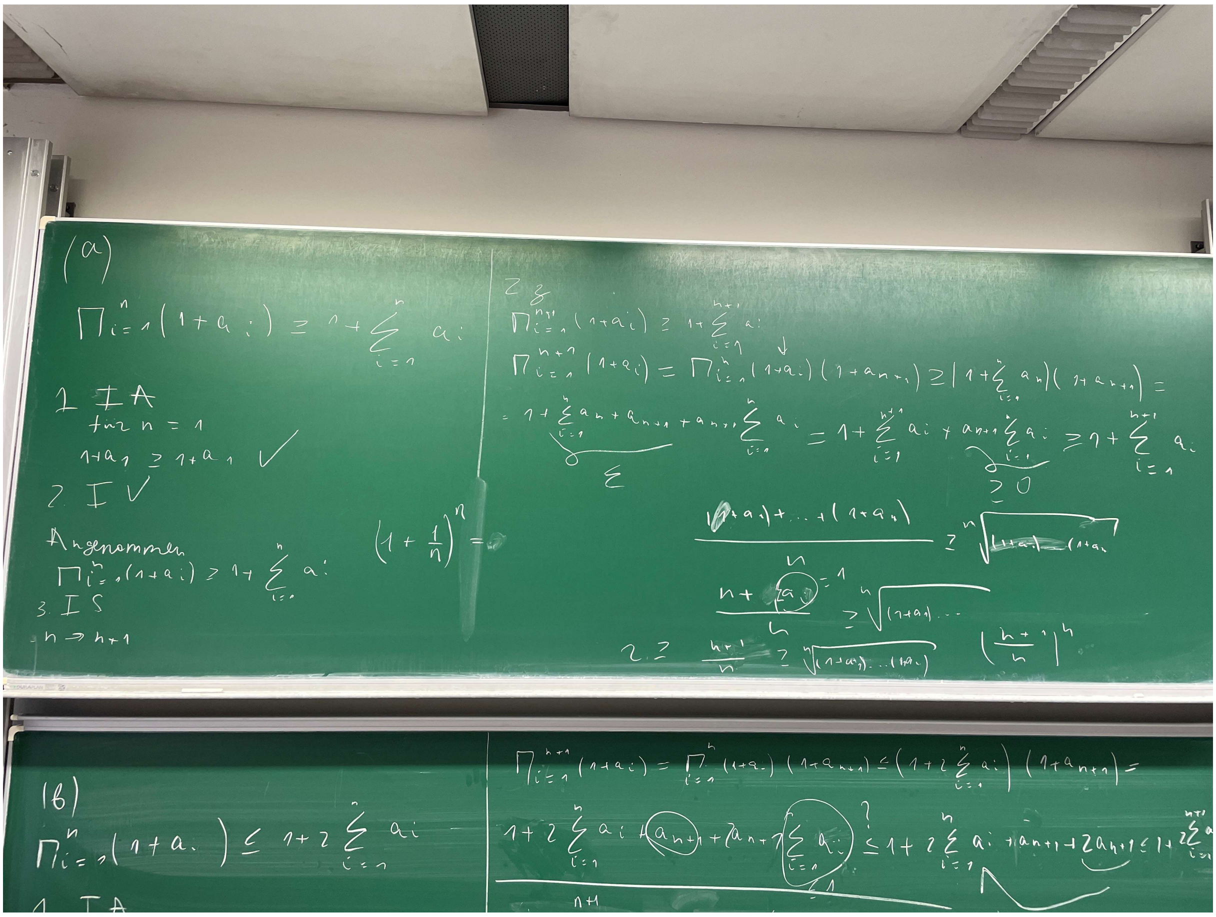
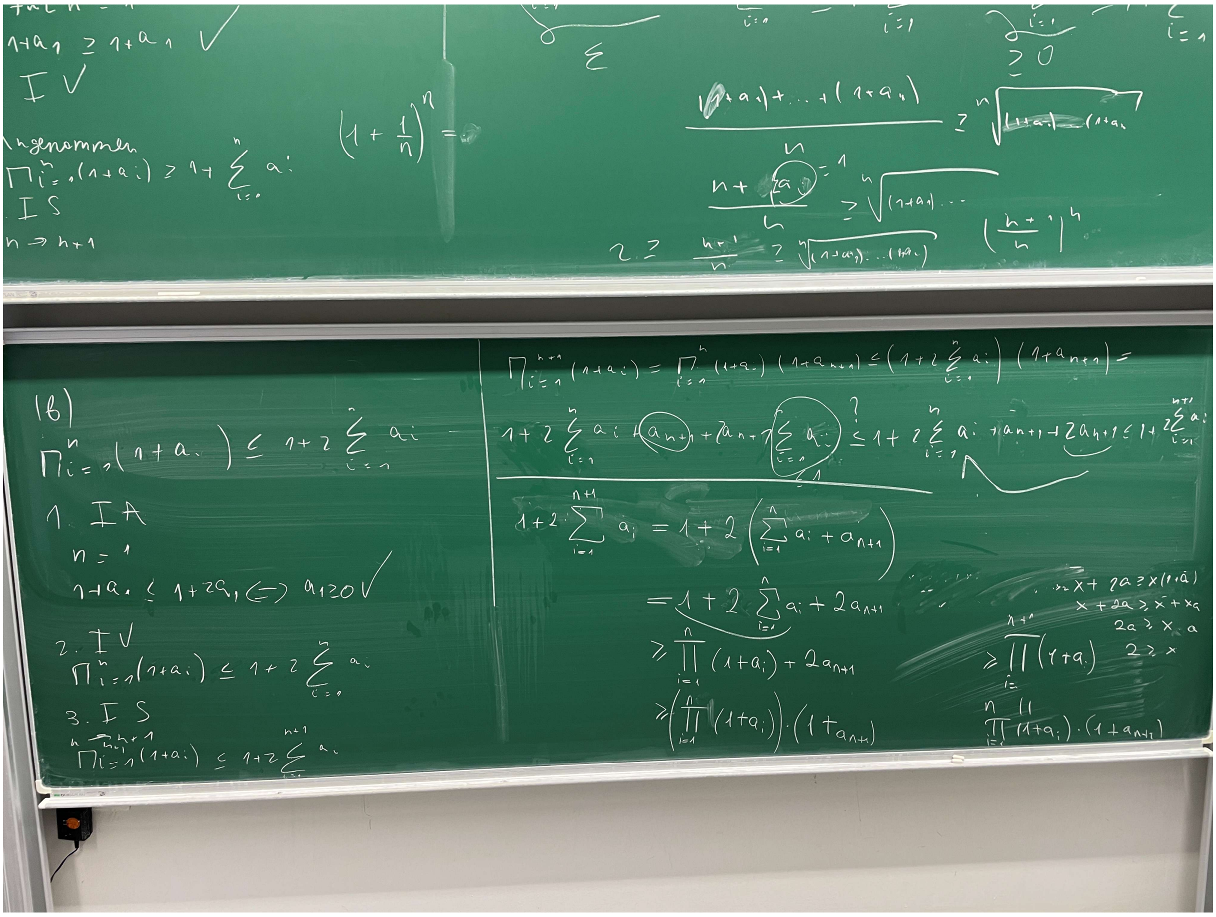


(a)
$\prod_{i=1}^{n}(1+a_i) \geq 1+\sum_{i=1}^{n} a_i$
1. IA
für $n = 1$
$1+a_1 \geq 1+a_1$ ✓
2. IV
Angenommen
$\prod_{i=1}^{n}(1+a_i) \geq 1+\sum_{i=1}^{n} a_i$
3. IS
$n \to n+1$
$\left(1+\frac{1}{n}\right)^n =$
z.z.
$\prod_{i=1}^{n+1}(1+a_i) \geq 1+\sum_{i=1}^{n+1} a_i$
$\prod_{i=1}^{n+1}(1+a_i) = \prod_{i=1}^{n}(1+a_i)(1+a_{n+1}) \geq \left(1+\sum_{i=1}^{n} a_i\right)(1+a_{n+1}) =$
$= 1+\sum_{i=1}^{n} a_i + a_{n+1} + a_{n+1}\sum_{i=1}^{n} a_i = 1+\sum_{i=1}^{n+1} a_i + a_{n+1}\sum_{i=1}^{n} a_i \geq 1+\sum_{i=1}^{n+1} a_i$
≥ 0
$\frac{(1+a_1)+\dots+(1+a_n)}{n} \geq \sqrt[n]{(1+a_1)\dots(1+a_n)}$
$\frac{n+\sum a_i}{n} \geq \sqrt[n]{(1+a_1)\dots}$
z.z. $\frac{n+1}{n} \geq \sqrt[n]{(1+a_1)\dots(1+a_n)}$
$\left(\frac{n+1}{n}\right)^n$
(b)
$\prod_{i=1}^{n}(1+a_i) \leq 1+2\sum_{i=1}^{n} a_i$
$\prod_{i=1}^{n+1}(1+a_i) = \prod_{i=1}^{n}(1+a_i)(1+a_{n+1}) \leq \left(1+2\sum_{i=1}^{n} a_i\right)(1+a_{n+1}) =$
$1+2\sum_{i=1}^{n} a_i + a_{n+1} + 2a_{n+1}\sum_{i=1}^{n} a_i \overset{?}{\leq} 1+2\sum_{i=1}^{n} a_i + a_{n+1} + 2a_{n+1} \leq 1+2\sum_{i=1}^{n+1} a_i$



(b)
$\prod_{i=1}^{n}(1+a_i) \leq 1+2\sum_{i=1}^{n} a_i$
1. IA
$n = 1$
$1+a_1 \leq 1+2a_1 \Leftrightarrow a_1 \geq 0$ ✓
2. IV
$\prod_{i=1}^{n}(1+a_i) \leq 1+2\sum_{i=1}^{n} a_i$
3. IS
$n \to n+1$
$\prod_{i=1}^{n+1}(1+a_i) \leq 1+2\sum_{i=1}^{n+1} a_i$
$1+2\sum_{i=1}^{n+1} a_i = 1+2\left(\sum_{i=1}^{n} a_i + a_{n+1}\right)$
$= 1+2\sum_{i=1}^{n} a_i + 2a_{n+1}$
$\geq \prod_{i=1}^{n}(1+a_i) + 2a_{n+1}$
$\geq \left(\prod_{i=1}^{n}(1+a_i)\right)\cdot(1+a_{n+1})$
$x+2a \geq x(1+a)$
$x+2a \geq x+xa$
$2a \geq x\cdot a$
$2 \geq x$
$\geq \prod_{i=1}^{n+1}(1+a_i)$
$\prod_{i=1}^{n}(1+a_i)\cdot(1+a_{n+1})$

(c)

$$\prod_{i=1}^{n}(1-a_i) \geq 1-\sum_{i=1}^{n} a_i$$

1. IA

$n=1$

$1-a_1 \geq 1-a_1$ ✓

2. I ✓

$$\prod_{i=1}^{n}(1-a_i) \geq 1-\sum_{i=1}^{n} a_i$$

3. IS

$n \to n+1$

$$\prod_{i=1}^{n+1}(1-a_i) \geq 1-\sum_{i=1}^{n+1} a_i$$

$$\prod_{i=1}^{n+1}(1-a_i) = \prod_{i=1}^{n}(1-a_i)(1-a_{n+1}) \geq \left(1-\sum_{i=1}^{n} a_i\right)(1-a_{n+1}) =$$

$$= 1-\sum_{i=1}^{n} a_i - a_{n+1} + a_{n+1}\sum_{i=1}^{n} a_i = 1-\sum_{i=1}^{n+1} a_i + \underbrace{a_{n+1}\sum_{i=1}^{n} a_i}_{\geq 0} \geq 1-\sum_{i=1}^{n+1} a_i$$

$$1+2\sum_{i=1}^{n} a_i + a_{n+1} + (2a_{n+1})\cdot\sum_{i=1}^{n} a_i$$

$$1+a_{n+1}+2a_{n+1}\sum_{i=1}^{n} a_i + a_{n+1} - a_{n+1} =$$

$$= 1+a_{n+1}\left(2\sum_{i=1}^{n+1} a_i - 1\right) \leq 1+a_{n+1}\left(2\sum_{i=1}^{n+1} a_i - \sum_{i=1}^{n+1}\right) = 1+a_{n}$$