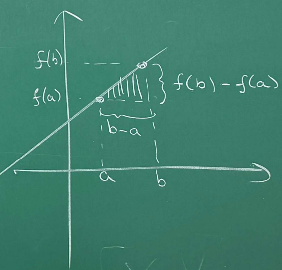
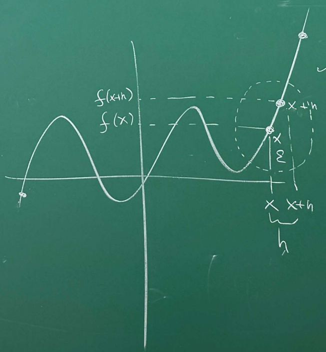
$$\frac{f(b)-f(a)}{b-a}$$

$(a,b) \in \mathbb{R}^2$



$$\frac{y_2-y_1}{x_2-x_1}$$



$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

1) a) $\lim\limits_{x \to 0} x \cot x = \lim\limits_{x \to 0} x \frac{\cos(x)}{\sin(x)} = \lim\limits_{x \to 0} \cos x \frac{x}{\sin x}$

$$= \lim_{x \to 0} \cos x \left(\left(\frac{x}{\sin x} \right)^{-1} \right)^{-1} = \lim_{x \to 0} \left(\frac{\sin x}{x} \right)^{-1} \cos x$$

$$= \lim_{x \to 0} \frac{\cos x}{\frac{\sin x}{x}} = 1$$

b) $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right) = \lim_{x\to 0}\left(\frac{\cos x}{\sin x} - \frac{1}{x}\right) = \lim_{x\to 0}\left(\frac{x\cos x - \sin x}{x\sin x}\right)$

Anwenden: Satz 10.18 a)

1. Ableitungen betrachten: Zähler: $(x\cos x - \sin x)' = -x\sin x \to 0$

   Nenner: $(x\sin x)' = \sin x + x\cos x \to 0$ ↯

2. Ableitungen: Zähler: $(-x\sin x)' = -\sin x - x\cos x \to 0$

   Nenner: $(\sin x + x\cos x)' = 2\cos x - x\sin x \to 2$

$\Rightarrow$ geht gegen 0

$(a,b) \in \mathbb{R}^2$

$$T_f(x,a) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$f(a+h) = f(a) + f'(a)h + \frac{1}{2}f''(a)h^2 + \mathcal{O}(h^2)$$

$$f(a-h) = f(a) - f'(a)h + \frac{1}{2}f''(a)h^2 + \mathcal{O}(h^2)$$

$$\frac{f(a+h)-2f(a)+f(a-h)}{h^2} = \frac{\cancel{2f(a)} + f''(a)h^2 - \cancel{2f(a)} + \mathcal{O}(h^2)}{h^2}$$

$$= \frac{f''(a)h^2 + \mathcal{O}(h^2)}{h^2}$$

$$= f''(a) + \frac{1}{h^2}\mathcal{O}(h^2) \xrightarrow[h\to 0]{} f''(a)$$

A3: $I \subseteq \mathbb{R}$ offenes Intervall, $f: I \to \mathbb{R}$ stetig und für ein $x_0 \in I$ f auf $I \setminus \{x_0\}$ diffbar

$$c := \lim_{x \to x_0} f'(x) = \lim_{x \to x_0} \left(\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \right) \underset{f \text{ stetig}}{=} \lim_{h \to 0} \left(\frac{f(\lim_{x \to x_0} x + h) - f(\lim_{x \to x_0} x)}{h} \right) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} = f'(x_0)$$

Sei $\varepsilon > 0$. n.V. $\delta > 0$: $0 < |x - x_0| < \delta \implies |f'(x) - c| < \varepsilon$

$$\implies \frac{f(x) - f(x_0)}{x - x_0} = f'(\xi), \text{ mit } \xi \in (x, x_0)$$

$$\implies \left| \frac{f(x) - f(x_0)}{x - x_0} - c \right| < \varepsilon$$

$$\frac{x \}\to 0}{\sin x \}\to 0}$$

t

$\int$

$$\Rightarrow \frac{f(x)-f(x_0)}{x-x_0} = f'(\xi) \quad , \text{ mit } \xi \in (x, x_0)$$

$$\Rightarrow \left| \frac{f(x)-f(x_0)}{x-x_0} - c \right| < \varepsilon$$

$$\frac{x \to 0}{\sin x \to 0}$$

$$\int_0^t \cos(x) \cdot dx$$